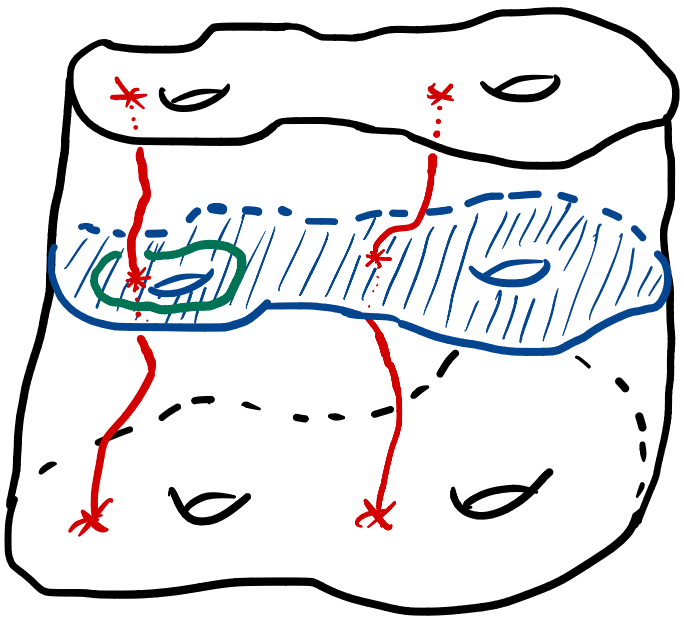
CFT/TFT correspondence beyond semisimplicity

Vienna, 17.04.2024, Aaron Hofer



Outline:

i) Motivation

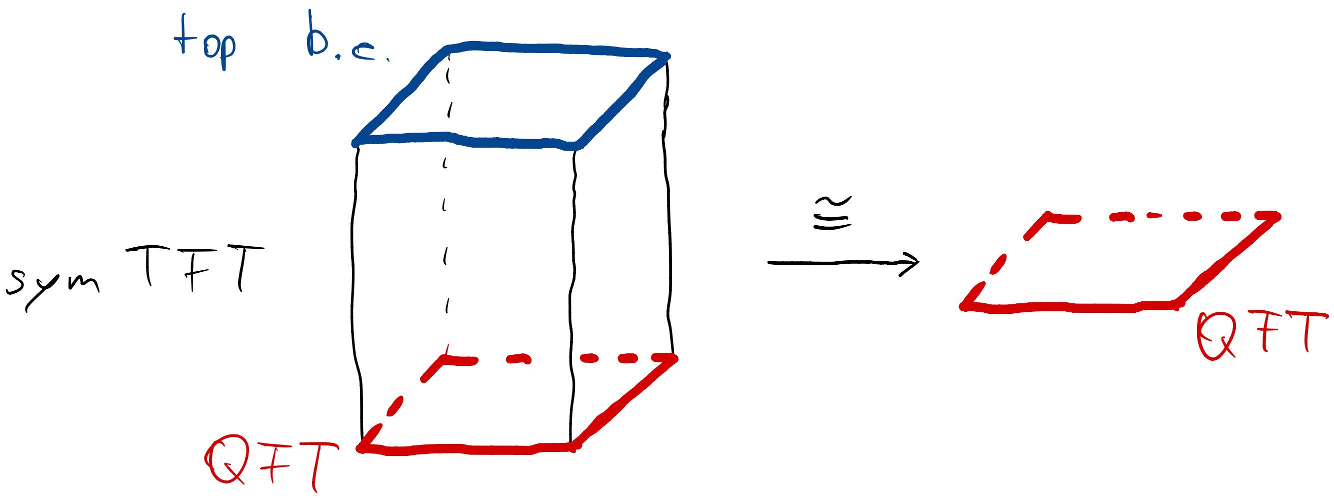
ii) CFT crashcourse

iii) FRS - construction

iv) Beyond semisimplicity

i) Motivation

Recently a popular perspective on quantum field theories has emerged, namely that of Symmetry TFT's. In this framework the symmetries of a d-dim QFT are encoded in a $(d+1)$-dim TFT with topological "Dirichlet" boundary condition:

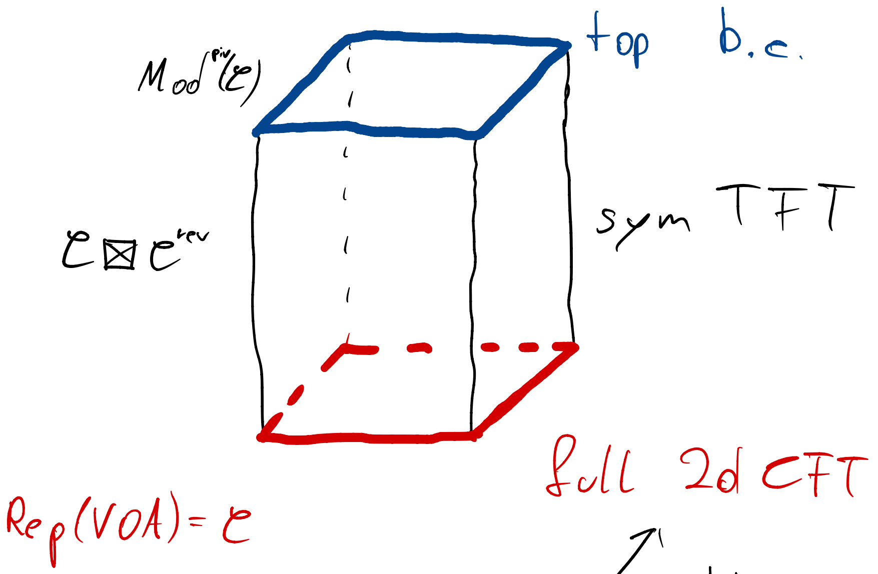


Can we understand this in a precise way?

Yes, for certain classes of 2d CFT's!

i) Motivation

CFT–TFT correspondence:



What is this mathematically?

semisimple	finite non-s.s.	derived
✓	?	??
✓	✓	??
MF ✓ Cor ✓ fields ✓	MF ✓ Cor ? fields ?	MF ✓ Cor ?? fields ??

Working Hypothesis: vertex operator algebra ⟶ sym. TFT
+ modular functor ⟵ conjecture
+ field content
+ correlators ⟵ FRS-construction + top. boundary condition

ii) CFT's

For 2d CFT's we expect the following mathematical structure:

1) chiral symmetry algebras are described as vertex operator algebras (VOA's)

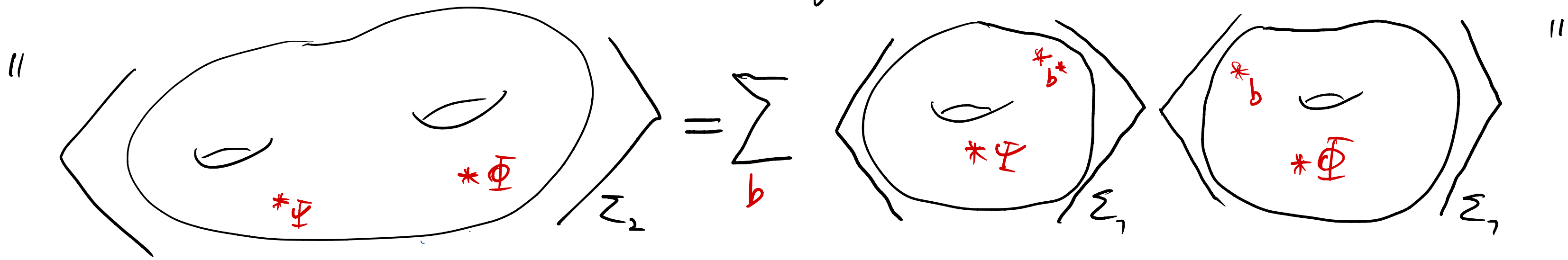
2) for any Riemann surface we get a conformal block space $\Sigma \mapsto Bl(\Sigma)$

3) field operators $\in$ VOA-modules which assemble into "nice" category

4) correlators of full CFT are elements of block space of complex double

$$Cor(\Sigma) \in Bl(\hat{\Sigma})$$

5) nice behaviour under cutting of surfaces (factorisation)



6) correlators should be invariant under mapping class group actions (related to single valuednes)

ii) CFT's

This can be packaged neatly using (higher) categorical language and TFTs:

Def A full modular functor is a symmetric monoidal 2-functor

$$Bl : \mathrm{Bord}^{or,oc}_{2+\varepsilon,2,1} \longrightarrow \mathrm{Prof}^{lex}_{\mathbb{K}}$$

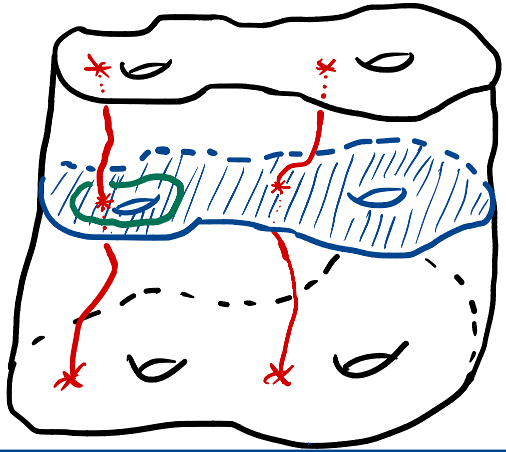
0:	intervalls & circles	finite $\mathbb{K}$-linear ab. cats
1:	surfaces	left exact profunctors $A \nrightarrow B \Leftrightarrow A^{op} \boxtimes B \xrightarrow{lex} \mathrm{vect}$
2:	diffeo's / isotopy	natural trafo's
$\diamond$:	gluing	convolution (coend $(G \diamond F)(-,\sim) := \oint^{B \in \mathcal{B}} G(B,\sim) \otimes_{\mathbb{K}} F(-,B)$)
$\circ$:	composition	composition
$\square$:	disjoint union	Deligne $\boxtimes$

Rem i) There is also a complex-analytic version of modular functors and it is conjectured to be "the same" as the topological one given above under certain conditions.

ii) For V a "finite enough" VOA there is an associated complex-analytic modular functor.

iii) FRS-construction

There is a notion of TFT with defects, where submanifolds of various codimension are decorated using extra data.



Thm] [Reshetikhin-Turaev, Carqueville-Schaumann-Runkel, Koppen-Mulevičius-Schweigert-Runkel]

Let $\mathcal{C}$ be a modular fusion cat. There exists a TFT with defects

$$Z_{\mathcal{C}} : \mathrm{Bord}^{\mathrm{def}}_{3,2}(\mathbb{D}_{\mathcal{C}}) \longrightarrow \mathrm{Vect}$$

constructed from $\mathcal{C}$.

Rem.] The TFT $Z_{\mathcal{C}}$ induces a modular functor $\mathrm{Bl}_{\mathcal{C}}$. For $\mathcal{C} = \mathrm{Rep}(V)$ with V a rational VOA it is conjectured that $\mathrm{Bl}_{\mathcal{C}} \cong \mathrm{Bl}_V$.

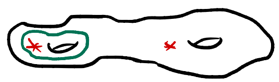
iii) FRS-construction

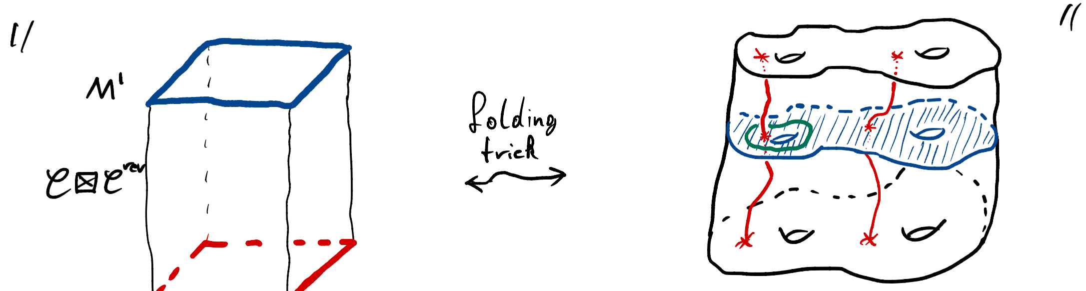
Main idea: Let Σ be a closed oriented surface with point defects, then

$Cor(\Sigma) \in Bl_{\mathcal{C}}(\hat{\Sigma}) \cong Z_{\mathcal{C}}(\hat{\Sigma})$ with $\hat{\Sigma} = \Sigma \sqcup -\Sigma$.

Q: Can we find a bordism $\emptyset \xrightarrow{M_\Sigma} \hat{\Sigma}$ such that $Z_{\mathcal{C}}(M_\Sigma)$ satisfies the conditions of a correlator?

[FRS]: Yes! But we need surface defects $M \in Mod^{tr}(\mathcal{C})$ as extra input.

e.g. $\Sigma =$ 



Apply Z $\rightsquigarrow$ $Cor_\Sigma^M \in Z(\Sigma) \otimes Z(\Sigma)^*$

$M_\Sigma = \Sigma \times I$ with surface defect M at $\Sigma \times \{0\}$.

Thm [Fuchs-Runkel-Schweigert]
Defining Cor_Σ^M as above gives consistent correlators for any Σ.

Rmk For surfaces with boundaries more care is needed.

iv) beyond semisimplicity

Why are non-semisimple theories interesting?

in physics:

- Applications in statistical physics, e.g. critical dense polymers
- WZW-models with supergroup target are often non-semisimple.
- Twists of SUSY QFT's usually non-semisimple/derived.

in mathematics:

- Many 2d TFT's are non-semisimple
- There are constructions of non-semisimple CFT's, can we understand them from a 3d perspective?
- Possibly stronger invariants
- Topological interpretation of algebraic structure
- Step towards derived TFTs

iv) beyond semisimplicity

Thm [De Renzi – Gainutdinov – Geer – Patureau-Mirand – Runkel]
Let $\mathcal{C}$ be a finite modular tensor category then there exists a (non-compact) TFT with line defects

$$\mathcal{Z}_{\mathcal{C}} : \mathrm{Bord}_{3,2}^{n.c}(\mathcal{C}) \longrightarrow \mathrm{Vect}_{\mathbb{K}}.$$

Thm [H-Runkel; DGGPR]
The above TFT induces a full modular functor

$$Bl_{\mathcal{C}} : \mathrm{Bord}_{2+\varepsilon,2,1}^{or,oc} \longrightarrow \mathrm{Prof}_{\mathbb{K}}^{\mathrm{lex}}$$

Can we apply the FRS-construction to non-semisimple theories?
In principle yes, however there are two technical difficulties:

i) The non-semisimple TQFT's of [DGGPR] are without surface defects.
What is the correct algebraic datum? $\rightsquigarrow$ Pivotal module categories

ii) The operation of doubling a surface with point defects becomes very subtle for non-semisimple labeling data.

Possible solution using a reformulation with 2-categorical language:

iv) beyond semisimplicity

Can we formulate FRS functorially?

conj. Def. A consistent system of correlators is a symmetric monoidal oplax natural transformation

$$\begin{tikzcd} \mathrm{Bord}^{del,o.c.}_{2+\varepsilon,2,1}(\mathbb{D}) \arrow[r, bend left=50, "\Delta_{\mathbb{K}}"] \arrow[r, bend right=50, "BL"'] \arrow[r, phantom, "\Downarrow \mathrm{Cor}"] & \mathrm{Prof}^{lex}_{\mathbb{K}} \end{tikzcd} .$$

$\Delta_{\mathbb{K}}$ ← constant profunctor to $\mathbb{K}$

$(2,1)$-category of top. world sheets

BL ← full modular functor

Rem Very similar to relative TFTs of [FT, JFS] but not fully extended.

Conjecture The FRS-construction corresponds to the following composition

$$\begin{tikzcd} \mathrm{Bord}^{del,o.c.}_{2+\varepsilon,2,1}(\mathbb{D}) \arrow[r, bend left=50, "\emptyset"] \arrow[r, bend right=50, "\widehat{(-)}"'] \arrow[r, phantom, "\Downarrow M_-"] & \mathrm{Bord}^{del}_{3,2,1}(\mathbb{D}) \arrow[r, "Z"] & \mathrm{Prof}^{lex}_{\mathbb{K}} \end{tikzcd}$$

$\emptyset$ ← trivial 2-functor

connecting bordism

$\widehat{(-)}$ ← orientation double functor

Problem: i) $\mathrm{Bord}^{del}_{3,2,1}(\mathbb{D})$ is very subtle to define.

ii) Construction of $\mathrm{Bord}^{del}_{3,2,1}(\mathbb{D}) \xrightarrow{Z} \mathrm{Prof}^{lex}_{\mathbb{K}}$ approachable only for separable defects

Thank you!